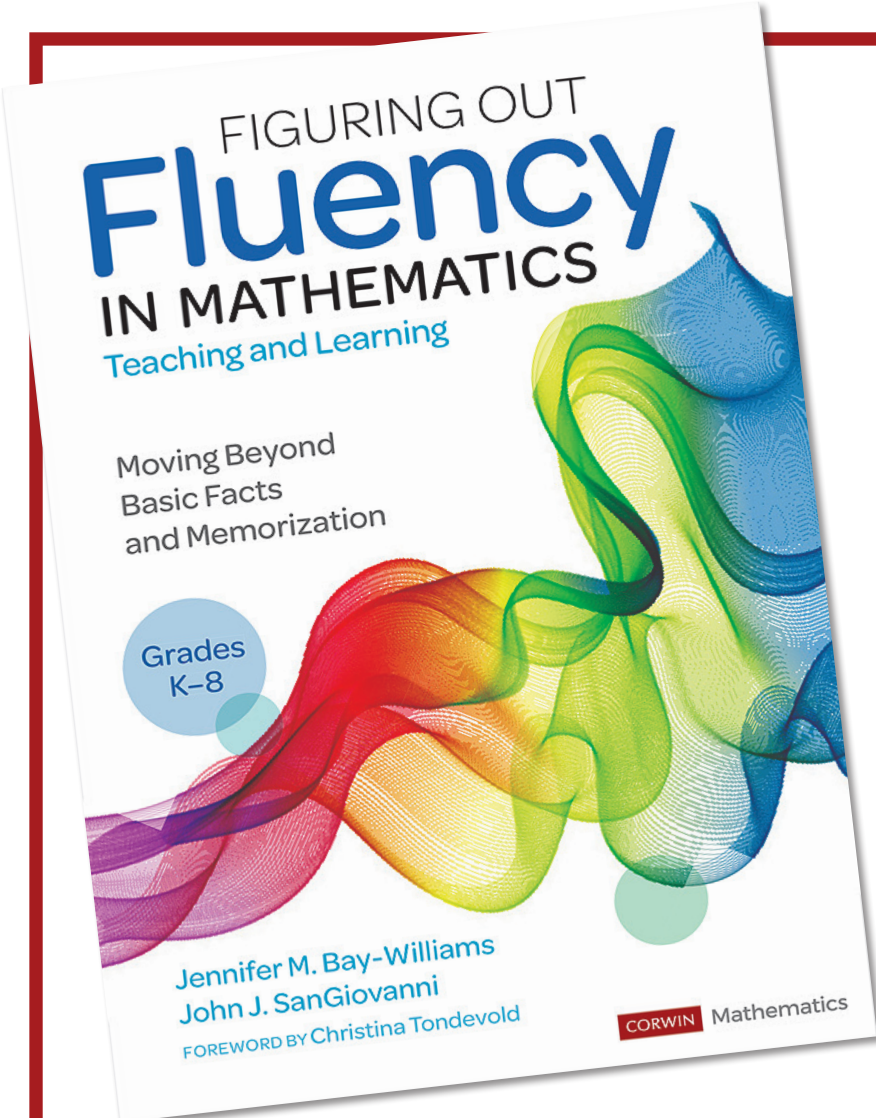

Thank you

FOR YOUR INTEREST IN CORWIN

An important aspect of developing mathematical fluency is checking for reasonableness. Here is a process and activity to help your students develop this skill.

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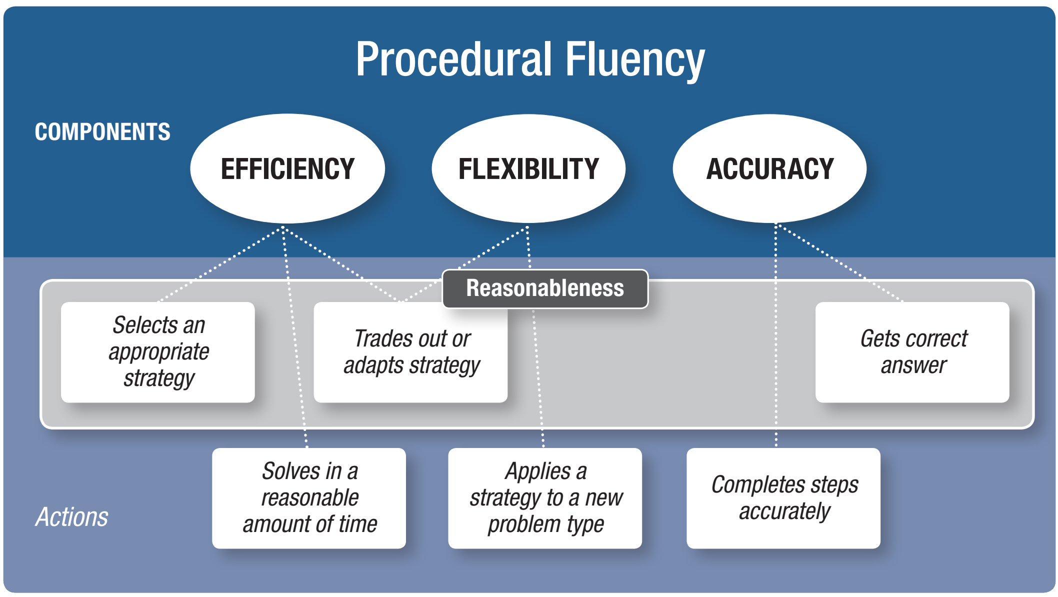
CHECKS FOR REASONABLENESS

Have you ever caught yourself trying to mentally solve a problem using a traditional algorithm, only to think later to yourself, Oh, I could have done that so much more simply! Guiding students through the six Fluency Actions as they solve problems, there is (or should be) a voice in their head asking and saying things like, "Is there a shorter method? This seems to be going nowhere," and "Does this answer make sense?" Fluency includes checks for reasonableness throughout the process of solving the problem. Figure 1.5 layers reasonableness as part of the comprehensive description of procedural fluency.

FIGURE 1.5 ● Procedural Fluency Components, Actions, and Checks for Reasonableness



Source: Adapted with permission from D. Spangler & J. Wanko (Eds.), *Enhancing Classroom Practice with Research behind Principles to Actions*, copyright 2017, by the National Council of Teachers of Mathematics. All rights reserved.

Put more simply and in language that is student-friendly, here are the three opportunities to check for reasonableness:

Choose. Choose a strategy that is efficient based on the numbers in the problem.

Change. Change the strategy if it is proving to be overly complex or unsuccessful.

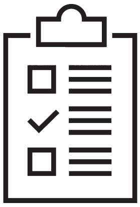
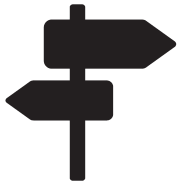
Check. Check to make sure the result makes sense.

These are quick actions that frame a metacognitive conversation but are rarely explicitly taught or recognized. Teaching and reinforcing these reasonableness checks with your students will greatly

TEACHING TAKEAWAY

Explicitly teaching the Choose, Change, Check metacognitive process for checking reasonableness will help students develop fluency and confidence in themselves.

FIGURE 1.6 ● Choose, Change, Check Reflection Card for Students

CHECKS FOR REASONABLENESS		
Choose	Change	Check
		
Is this something I can do in my head? What strategy makes sense for these numbers?	Is my strategy going well, or should I try a different approach? Does my answer so far seem reasonable?	Is my answer close to what I anticipated it might be? How might I check my answer?

Icon sources: Choose by iStock.com/Enis Aksoy; Change by iStock.com/Sigit Mulyo Utomo; Check by iStock.com/Indigo Diamond

aid in their fluency. Explicitly teaching the Choose, Change, Check metacognitive process for checking reasonableness can help students develop fluency and confidence in themselves. One way to explicitly attend to reasonableness is to provide students with Question Cards (see Figure 1.6). Students have these cards for reference as they think through problems individually, with a partner, or a small group.

In the Common Core State Standards (CCSS) Mathematical Practices (MP), reasonableness is addressed in both MP1—*Make sense and persevere*—and MP8—*Look for and express regularity in repeated reasoning* (National Governors Association Center for Best Practices and Council of Chief State School Officers [NGA Center & CCSSO], 2010):

> MP1: Mathematically proficient students . . . plan a solution pathway rather than simply jumping into a solution attempt . . . monitor and evaluate their progress and change course if necessary . . . [and] check their answers to problems . . . continually ask[ing] themselves, "Does this make sense?"
>
> MP8: Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. . . . As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Reasonableness is certainly underemphasized in standards documents. In the CCSS, beyond the mention in MP8, reasonableness is mentioned in only one standard at Grades 3, 4, 5, and 7. The other grades have no mention of it. Yet the questions like "Is there a shorter method?" and

ACTIVITY 1.3
ROUTINE: "IS IT REASONABLE?"

Materials: Three "________ is about ________" statements (see examples in the following chart)

Directions: Pose the first statement. Give students a cue for *Reasonable* and *Not Reasonable*.

For example, you might use sign language, with  for reasonable and  for not reasonable. Prompt students to make a private decision and wait for a "Show Me" request. Students share their decision and discuss why. Alternatively, small groups can discuss which are reasonable or not and then share with whole class.

REASONABILITY STATEMENTS		
Subtraction Within 1,000	**Multiplication With Decimals**	**Percentage**
985 – 328 is about 600	2.56 × 4 is about 10	$\frac{13}{40}$ is about 33%
549 – 98 is about 300	13.44 × 2.88 is about 26	$\frac{11}{24}$ is about 33%
671 – 443 is about 300	4.75 × 5 is about 25	$\frac{17}{30}$ is about 60%

Source: signs for R and N by iStock.com/Jayesh

"Does this answer make sense?" are clearly essential to doing mathematics. Infusing reasonableness into the curriculum is largely the responsibility of the teachers and leaders who design lessons, units, and curriculum. Routines are effective for reinforcing such underemphasized skills. Activity 1.3 contains one idea to add to your routine repertoire to help students practice checking for reasonableness.

The "Is It Reasonable" routine in Activity 1.3 helps students develop ways to check for reasonableness. But if reasonableness is limited to a routine, students won't develop the metacognitive practice of thinking "is this reasonable" as they are solving problems embedded in their classroom tasks or homework. As indicated in Figure 1.5, three of the six Fluency Actions include attending to reasonableness. Importantly, these reasonableness Fluency Actions occur before you start solving a problem, during the solving, and at the conclusion. Developing procedural fluency, then, includes helping students develop the metacognitive practices throughout solving a problem. One way to do this is to model "Ask-Yourself Questions" (see Figure 1.7). Ask-Yourself Questions are initially modeled by the teacher to make such thinking visible to students, with the intent that students will internalize the questions as they solve problems independently (Kelemanik et al., 2016).

Stop & Reflect

How might you infuse these Ask-Yourself Questions into your classroom or school?

FIGURE 1.7 ● Reasonableness Ask-Yourself Questions

CHOOSE

Related Fluency Action: Selects an appropriate strategy

Before you solve, ask yourself these questions:

- *Is this something I can do in my head?*
- *Is the strategy or method I am considering a reasonable approach for the numbers in this problem?*
- *Is it reasonable to use the standard algorithm for this problem (or is there a shorter method)?*
- *What is a good estimate for the answer?*

CHANGE

Related Fluency Action: Trades out or adapts strategy

During the solving, ask yourself these questions:

- *Is this answer I got [partway through a process] reasonable?*
- *Is this amount of 'mess' reasonable, or did I make a mistake or pick a bad method?*
- *Should I trade out my strategy?*
- *How might I adapt my strategy?*

CHECK

Related Fluency Action: Gets correct answer

After solving, ask yourself these questions:

- *Is this answer close to what I anticipated it might be?*
- *Does my answer make sense?*
- *How can I check to see if my answer is correct?*

Did you notice that in the CCSS Mathematical Practices and in the Ask-Yourself Questions, "reasonable" occurs in three phases of solving a problem (not just at the end)? Reasonableness throughout the problem-solving process must be modeled and discussed frequently. You can use the Ask-Yourself Questions to craft anchor charts to support students as they develop aspects of reasonableness. That is not to suggest that you stop students at three points to ask them to check for reasonableness, but rather to have reasonableness embedded in the process of solving a problem. If they only check their answer at the end, it can be too late—they may have spent an unnecessary amount of time with an approach that was not a good method in the first place.

So when students choose a strategy, we want them to think about that strategy being a good fit or a good idea for solving that specific problem. Is the strategy reasonably useful or efficient? This takes practice. Activity 1.4 uses worked examples to focus on *choosing*. Students decide and discuss if the selected strategy was a good choice or not. The activity lends to journaling, independent work, or even homework and can be the focus of a rich classroom discussion that supports fluency and reasonableness (as there may be disagreements).

TEACHING TAKEAWAY

Put Ask-Yourself Questions on anchor charts to support students as they develop aspects of reasonableness.

ACTIVITY 1.4
FOCUS TASK: GOOD CHOICE OR BAD CHOICE

Materials: Set of problem(s), each with a strategy to critique (see examples in Figure 1.8)

Directions: Pose one problem, along with a strategy explanation. Give students a minute to decide if the strategy is a good choice or bad choice. First, ask students to explain what the student did. Second, have students tell if they think the strategy was a good choice or bad choice and why. Third, ask students to offer alternatives for the problems where they decide the example is a bad choice. *Note*: This is an efficiency discussion with students, and in the end, the purpose is to attend to choice, not to agree on an absolute answer.

FIGURE 1.8 ● Examples of Problems for Good Choice or Bad Choice Activity

EXAMPLES	STRATEGY	GOOD CHOICE OR A BAD CHOICE? WHY?
37 + 74	Jimmy counted up, by ones, from 74.	
4,260 ÷ 60	Zoe broke 4,260 apart into 4,200 + 60 and divided both parts by 60 and then added the answers back together.	
$\frac{3}{8}+2\frac{4}{16}$	Brendan converted $2\frac{4}{16}$ to $\frac{36}{16}$, changed $\frac{3}{8}$ to $\frac{6}{16}$, and added to get $\frac{42}{16}$. Then, he changed $\frac{42}{16}$ to a mixed number.	

This activity grows into comparing two worked examples, which is highly effective in supporting student development of flexibility, a key Fluency Action (Rittle-Johnson et al., 2009).

While reasonableness is something to attend to more intentionally, there are some practices to avoid:

1. Don't ask about reasonableness without first building a concept of what reasonable means.
2. Don't treat reasonableness as another step of a procedure.
3. Don't try to use a "standard algorithm" for checking for reasonableness. In other words, applying the inverse operation to a problem may check one's accuracy with a procedure but does not necessarily determine if the results are reasonable. For example, see this student's thinking: The original problem was 5.25 ÷ .25. She divided and moved the decimal point. To check her work, she multiplied and moved the decimal down. In both cases, her work seems related to lining up decimals for addition and subtraction.